Energy Confinement Time in a Magnetically Confined Thermonuclear Fusion Device

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Presented at 2022 Sherwood Fusion Theory Conference

Abstract

The single most important scientific question in fusion research may be confinement in a fusion plasma. A recently-developed theoretical model is reviewed for the confinement time of ion kinetic energy in a material where fusion reactions occur. In the theoretical model where ion stopping was considered as a key mechanism for ion kinetic energy loss, an estimate was obtained for the confinement time of ion kinetic energy in a warm-dense D-T plasma. As ions transfer their kinetic energies to electrons via ion stopping and thermalization between the ions and the electrons takes place, spontaneous electron cyclotron radiation is identified as a key mechanism for electron kinetic energy loss in a magnetically confined fusion plasma. The energy confinement time obtained analytically is found in agreement with measurements from TFTR and Wendelstein 7-X. An advanced Lawson criterion is derived for a magnetically confined thermonuclear fusion reactor. A limit of fusion energy relative to supplied heating energy is predicted, which appears to be consistent with the latest fusion energy record achieved experimentally at JET.

Introduction

There has been a stream of new fusion energy records achieved recently in thermonuclear fusion experiments[1,2]. In magnetically confined fusion, it is believed that the same modeling used in achieving fusion energy records predicts that ITER will work[1]. However, the single most important scientific question may still be energy confinement time in a fusion plasma[3]. The energy confinement time of a plasma is defined as the thermal energy content of the plasma divided by the power loss,

$$\tau_E \equiv \frac{W}{P_{loss}}$$

where W is the thermal energy of the plasma, and P_{loss} is the power loss. Until the recent work[4], there has not been a simple theory for τ_E calculation. The fusion research community relies on derived empirical scaling laws for energy confinement times in fusion reactor designs[5].

Confinement Time of Ion Kinetic Energy in Neutral Materials and Warm Dense Plasmas

Neutral Materials: As an energetic ion traverses a material, it loses its kinetic energy on average via ion stopping, a process dominated by transferring ion kinetic energy to the electrons in the material. For a proton with a low energy (1-10 keV/ N_A), the ion stopping cross section ϵ is given by[6]

$$\frac{\epsilon}{10^{-15}\ \mathrm{eV}\cdot\mathrm{cm}^2} = A_1\sqrt{\frac{E_i}{\mathrm{keV}\cdot N_A}}$$

where N_A is the atomic mass number of the atom in the material, and A_1 is a fitting parameter. For hydrogen materials, A_1=1.262, the rate of ion kinetic energy loss is[7]

$$\frac{dE_i}{dt} = -\frac{E_i}{\tau_{E,i}} \text{ and } \frac{1}{n\tau_{E,i}} \equiv (10^{-15}\ \mathrm{eV}\ \cdot \mathrm{cm}^2)A_1\left(\frac{2}{\mathrm{keV}\cdot N_A\, m}\right)^{1/2}$$

where $\tau_{E,i}$ is the *confinement time of ion kinetic energy.* Because of ion stopping, it is generally impossible to generate net fusion energy by striking a neutral material such as a solid target of tritium, lithium or boron with an ion beam.

Table 1 $n\tau_{E,i}$ of Proton, Deuteron and Triton in Isotopes of Hydrogen[7]

Isotope	$n\tau_{E,i}$ (proton)	$n\tau_{E,i}$ (deuteron)	$n\tau_{E,i}$ (triton)
Hydrogen (H)	1.8×10^{16} s/ m^3	2.6×10^{16} s/ m^3	3.1×10^{16} s/ m^3
Deuterium (D)	1.8×10^{16} s/ m^3	2.6×10^{16} s/ m^3	3.1×10^{16} s/ m^3
Tritium (T)	1.8×10^{16} s/ m^3	2.6×10^{16} s/ m^3	3.1×10^{16} s/ m^3

Warm Dense D-T Plasmas: Ion stopping in hot plasmas is ill-defined because ions and electrons tend to thermalize. However, there are indications that ion stopping cross section in a warm-dense plasma[8,9] is comparable to that in a neutral material perhaps within one or two orders of magnitude. Under the assumption that ion stopping cross section in a plasma is comparable to that in a plasma, if we ignore effects of thermalization between the ions and electrons, we may crudely estimate the confinement time of ion kinetic energy in a D-T plasma by averaging $n\tau_{E,i}$ of deuteron in tritium and $n\tau_{E,i}$ of triton in deuterium, i.e.,

$$n\tau_{E,i}(\mathrm{D-T\ plasma}) \approx 3\times10^{16}\ \mathrm{s/\ m^3},$$

which is a constant, independent of the target ion density or the ion kinetic energy in the range of 2 keV to 30 keV[7].

Confinement Time of Electron Kinetic Energy

An isothermal electron gyrating in a magnetic field, B, loses its kinetic energy E_e via cyclotron radiation emission at a rate of

$$\frac{dE_e}{dt} = \frac{3dE_{e\perp}}{2dt} = -\frac{3E_{e\perp}}{2\tau_{E,e}} = -\frac{E_e}{\tau_{E,e}} \text{ and } \tau_{E,e} \equiv \frac{3c}{4\omega_c^2 r_e}$$

where c is the speed of light, $r_e = 2.8\times10^{-15}$ m is the classical electron radius, and $\omega_c = eB/m_e$ is the electron cyclotron frequency. At $B = 1$ T, $\tau_{E,e}$=2.6 s.

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Spontaneous electron cyclotron radiation losses dominate Rutherford-scattering bremsttrahlung radiation losses. The ions and electrons tend to thermalize such that $E_i \approx E_e = E$, where E_i is the total kinetic energy of an ion on the average. The energy loss rate of an ion is about the same of an electron. The confinement time of plasma thermal energy is simply given by[4]

$$\tau_E \equiv -\frac{1}{E}\frac{dE}{dt} = \tau_{E,e} = \frac{3c}{4\omega_c^2 r_e}$$

Comparison between Theory and Experiment

Table 2 Theory vs. Experiment[4]

Device	B (T)	τ_E (s) experiment	τ_E (s) theory
TFTR[3]	5.6	0.08	0.083
W-7X[10]	2.5	0.4	0.42

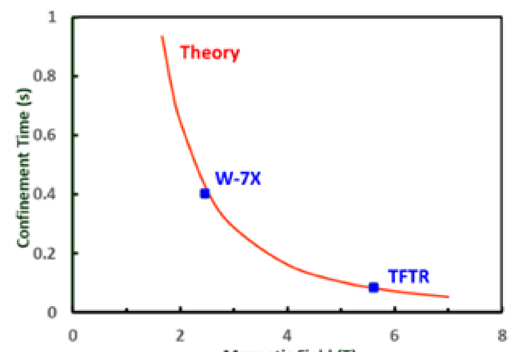

Fig. 1 Theory vs experiment.[4] TFTR (qualitative) W-7X (quantitative)

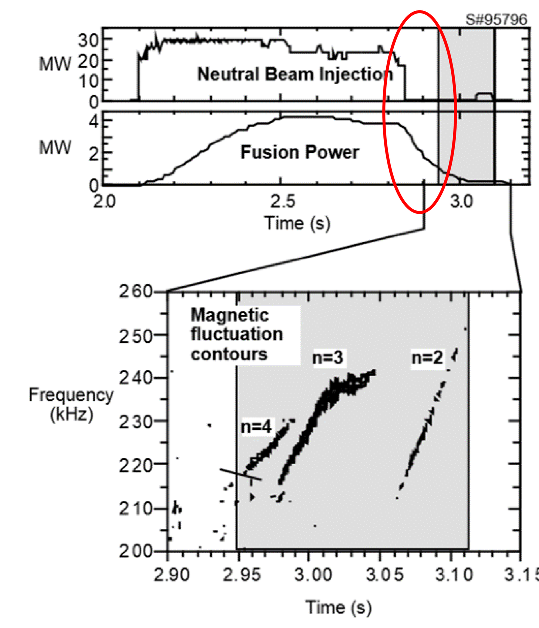

Fig. 2 Experimentally measured neutral beam heating and fusion power at TFTR.[3]

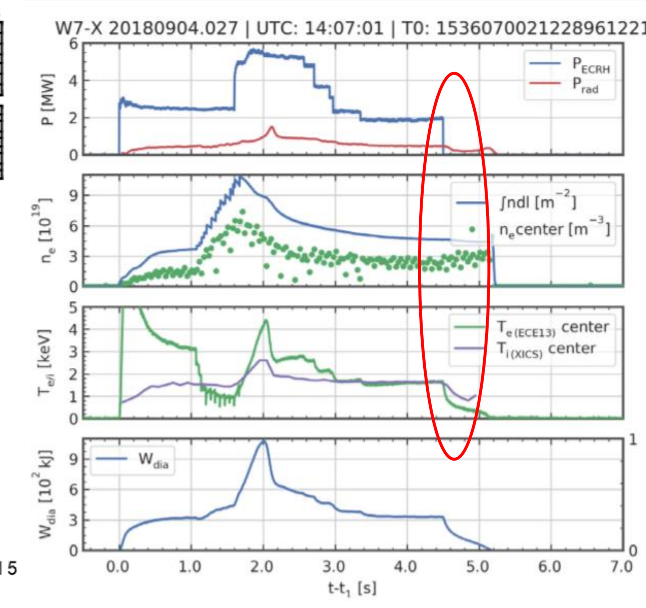

Fig. 3 High performance discharge with pellet injections in a hydrogen plasma at Wendelstein 7-X.[10]

Advanced Lawson Criterion

In a magnetically confined thermonuclear fusion reactor in a laboratory (such as ITER), the criterion for ignition in a D-T fusion reactor is

$$nk_BT\tau_E \geq 12(k_BT)^2/E_{ch}\,\langle\sigma v\rangle$$

where E_{ch}=3.4 MeV is the fusion-produced α-particle energy, and $\langle\sigma v\rangle$ is the fusion reactivity. At the minimum (or k_BT=14 keV), the criterion in terms of the mechanical and magnet pressures, p and p_{mag}, respectively, is simply[4]

$$\beta \equiv p/p_{mag} \geq 0.92$$

which is much greater than those achievable in past, present and planned ITER experiments in tokamak or stellarator configurations, but may be achievable in z- and theta-pinch configurations. Indeed, for a typical tokamak with a ratio of major radius to minor radius of $R/a = 3$, the MHD stability limit[5] is β=0.052.

Limit of Fusion Energy Gain

The limit of Q measuring fusion energy relative to supplied heating energy is

$$Q_{Limit}(\mathrm{TD\ fusion}) = \frac{\mathrm{maximun\ fusion\ energy}}{\mathrm{supplied\ heating\ energy}} \approx \frac{5\beta}{0.92} = 5.4\beta$$

which appears to be consistent with the latest fusion energy record with sustained Q = 0.33 for 5 s achieved experimentally at JET.[1]

Conclusion

A theory of energy confinement was presented in a magnetically confined thermonuclear fusion reactor. The confinement time of thermal energy was shown to be primarily determined by spontaneous electron cyclotron radiation. Excellent agreement was found between theory and experiment. An advanced Lawson criterion was derived for magnetically confined fusion reactors. A limit of fusion energy relative to supplied heating energy was predicted, which appears to be consistent with the latest fusion energy record achieved experimentally at JET.

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